DSP Lec 5

Quiz

$x(n) = \{1, 1, 1, 1\}$, $h(n) = \{2, 2\}$

Find $y(n) = x(n) * h(n)$
using inverse z-transform.

Sol

$$x(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$$

$$h(n) = 2\,\delta(n) + 2\,\delta(n-1)$$

$$Y(z) = X(z) * H(z)$$

$$= \left[1 + z^{-1} + z^{-2} + z^{-3}\right]\left[2 + 2z^{-1}\right]$$

$$Y(z) = 2 + 4z^{-1} + 4z^{-2} + 4z^{-3} + 2z^{-4}$$

$$\boxed{y(n) = 2\,\delta(n) + 4\,\delta(n-1) + 4\,\delta(n-2) + 4\,\delta(n-3) + 2\,\delta(n-4)}$$

* Discrete Fourier Transform (DFT)

⇒ if we want to analysis for any signal, we want to evaluate some parameters as:

1. freq. content.
2. Power and energy density.
3. Amplitude.
4. Periodicity.

→ in frequency domain

← في بعض الـ APPs. بنحتاج الحاجات دي .

مثلاً [illegible] transmition

useful for

↳ we want to convert signals from time domain to frequency domain to study some parameters of the signals as (frequency content and power and energy density), which couldn't be studied in time domain.

* Some Applications depend on analysis of the signals on frequency domain as filter design.

$$x(t) \xrightarrow{\text{Laplace}} X(s)$$

timedomain — s-domain

$$X(s) = \int_{0}^{\infty} x(t)\, e^{-st}\, dt$$

→ Continous Fourier transform (FT)

← إحنا مش بندرس ال (Continous) لكن دورنا نعرف إزاي حولنا من Cont. ل (discrete)

$$\boxed{X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$ → Continous Fourier transform

← مشكلة :- إنك متقدرش تستخدمه مع ال (digital systems)

← عشان كده لازم نقطع ال (signal) دي ونحولها ل (discrete)

$$X(\omega) \xrightarrow{\text{Discritization}} X(K)$$

ـــ تلخيص ما سبق :

$$x(t) \xrightarrow{FT} X(\omega)$$

$$x(n) \xrightarrow{DFT} X(K)$$

ـــ لو ال (signal) اللي بتحولها (Periodic)

في ال (time domain) و (frequency domain) هتنتج

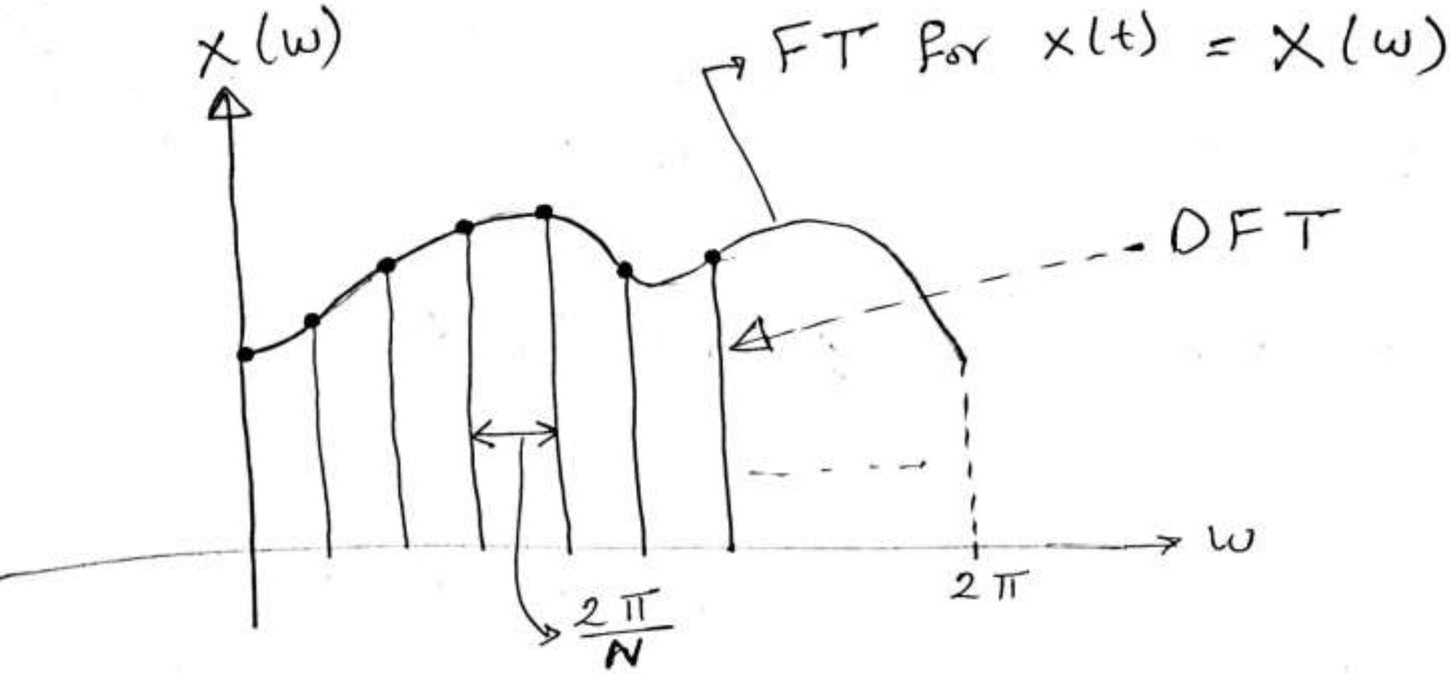
(Periodic signal) أيضاً.

$$\underset{\text{Periodic}}{\underline{x(t)}} \xrightarrow{FT} \underset{\text{Periodic}}{\underline{X(\omega)}}$$

and it's periodic every 2π interval of freq.

بتتكرر كل 2π

* if, we assume the signals we deal with are Periodic



$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \Rightarrow \text{F.T}$$

⇒ assume $X(\omega)$ is discretized to N samples

$$\underbrace{\omega}_{\text{Continous Freq.}} \longrightarrow \frac{2\pi}{N} K$$

$K \rightarrow$ the sample number for DFT.
$(0, 1, \cdots, N-1)$

$N \Rightarrow$ the no. of samples for D.F.T.

$n = 0, 1, 2, \ldots, N-1$
$K = 0, 1, 2, \ldots, N-1$

D.F.T

$$\boxed{X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-J\frac{2\pi}{N}kn}}$$

$n \Rightarrow$ the sample no. for discrete time sequence $x(n)$, $n = 0, 1, 2, \ldots, N-1$

← هل لازم الـ no. of sampling في X(K) يكون هو نفسه الـ (no. of sampling) لـ x(n)

← لأ مش لازم.

← لكن هنفترض إن (no. of sampling) واحد.

← بس ~~ممكن~~ يكون (no. of sampling) لـ X(K) أكبر من الـ (" " ") لـ x(n)

← بس لو أعرف no. of sampling = 4 لـ x(n)

و no. of sampling = 10 لـ X(K)

← هنحط الباقي أصفار ⇐ عشان كده بنفترض إنهم متساويين.

$\boxed{Ex}$ $x(n) = \{0, 1, 2, 3\}$, find the 4-point discrete Fourier transform of $x(n) \Rightarrow$ given $N = 4$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-J \frac{2\pi}{N} Kn}$$

<u>K = 0</u>

$$X(0) = \sum_{n=0}^{3} x(n) = x(0) + x(1) + x(2) + x(3)$$
$$= 6$$

<u>K = 1</u>

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-J \frac{2\pi}{4} n}$$
$$= x(0) + x(1)\, e^{-J\frac{\pi}{2}} + x(2)\, e^{-J\pi} + x(3)\, e^{-J\frac{3\pi}{2}}$$
$$= e^{-J\frac{\pi}{2}} + 2\, e^{-J\pi} + 3\, e^{-J\frac{3\pi}{2}}$$

<u>Note that</u>

$$e^{-J\theta} = \cos\theta - J\sin\theta$$

$$X(1) = \cos(90) - J\sin(90) + 2\left(\cos(180) - J\sin(180)\right)$$

$$+3\left(\cos(270) - J\sin(270)\right)$$

$$X(1) = -J - 2 + 3J = -2 + 2J$$

* $\underline{\underline{K = 2}}$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-J\frac{4\pi}{4}n} = \sum_{n=0}^{3} x(n)\, e^{-J\pi n}$$

$$= \underbrace{x(0)}_{0} + \underbrace{x(1)}_{1} e^{-J\pi} + \underbrace{x(2)}_{2} e^{-J2\pi} + \underbrace{x(3)}_{3} e^{-J3\pi}$$

$$= e^{-J\pi} + 2e^{-J2\pi} + 3e^{-J3\pi}$$

$$= \left(\cos(180) - J\sin(180)\right) + 2\left(\cos(360) - J\sin(360)\right)$$

$$+ 3\left(\cos(180) - J\sin(180)\right)$$

$$X(2) = -1 + 2 - 3 = -2$$

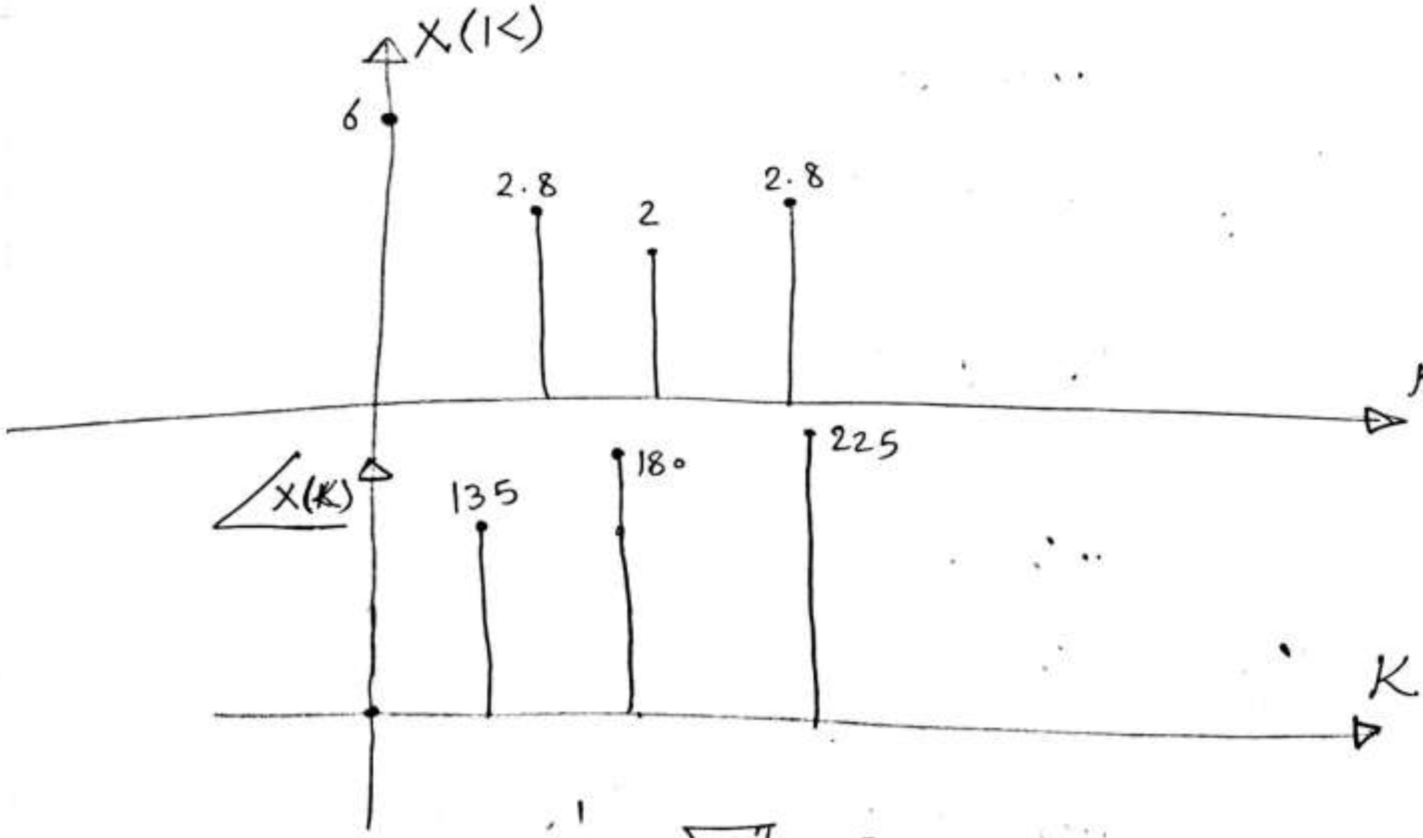
$\underline{\underline{K = 3}}$

$$X(3) = -2 - 2J$$

$$\underbrace{X(K)}_{\text{DFT for } x(n)} = \{6, -2+J2, -2, -2-2J\}$$

$$|X(K)| = \{6, 2\sqrt{2}, 2, 2\sqrt{2}\}$$

$$\angle X(K) = \{0, 135, 180, \underbrace{225}_{\text{or } -135}\}$$



ـــ في الإمتحان هيجيب $N=3$ or $N=4$

ـــ صورة الحل الماضية لو حليت بيها عادي لكن القادم حل أبسط ولك حرية الإختيار في الحل.

$$x(n) \xrightarrow{DFT} X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-J\frac{2\pi}{N}nK}$$

$$W_N = e^{-J\frac{2\pi}{N}}$$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, (W_N)^{Kn}$$

$K = 0, 1, 2, \dots, N-1$

$n = 0, 1, 2, \dots, N-1$

$$X_N = \begin{bmatrix} X(K=0) \\ X(K=1) \\ \vdots \\ X(N-1) \end{bmatrix}$$

$$x_N = \begin{bmatrix} x(n=0) \\ x(n=1) \\ \vdots \\ x(n=N-1) \end{bmatrix}$$

$$X_N = \begin{array}{c} \\ k=0 \\ \vdots \\ k=N-1 \end{array} \begin{array}{c} n=0 \qquad n=N-1 \\ \left[W_N^{kn} \right]_{N*N} \end{array} * x_N$$

For example $N = 4$

$$W_4 = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \begin{array}{c} \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} \\ \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^1 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \end{array}$$

$$W_4 = e^{-J\frac{2\pi}{4}} = \cos(90) - J\sin(90) = -J$$

$$W_4^0 = 1 \quad , W_4^2 = e^{-J\pi} = \cos(180) - J\sin(180) = -1$$

$$W_4^3 = e^{-J\frac{3\pi}{2}} = \cos(270) - J\sin(270) = J$$

$$W_4^4 = W_4^0 = 1 \quad , W_4^9 = W_4^1 = -J$$

$$W_4^6 = W_4^2 = -1$$

$$W_N^N = e^{-J\frac{2\pi}{N}N} = e^{-J2\pi} = 1$$

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -J & -1 & J \\ 1 & -1 & 1 & -1 \\ 1 & J & -1 & -J \end{bmatrix}$$

another solution

DFT for $x(n)$ $(N=4)$

$$X_4 = [W_4]\, x_4$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -J & -1 & J \\ 1 & -1 & 1 & -1 \\ 1 & J & -1 & -J \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 6 \\ -2+2J \\ -2 \\ -2-2J \end{bmatrix}$$

<u>For $N=3$</u> $\quad [W_3] = \left[W_3^{Kn} \right]_{3*3}$

$$= \begin{array}{c} \\ K=0 \\ K=1 \\ K=2 \end{array} \begin{array}{c} \begin{array}{ccc} n=0 & n=1 & n=2 \end{array} \\ \begin{bmatrix} W_3^0 & W_3^0 & W_3^0 \\ W_3^0 & W_3^1 & W_3^2 \\ W_3^0 & W_3^2 & W_3^4 \end{bmatrix} \end{array}$$

where $W_3 = e^{-J\frac{2\pi}{3}}$

$* \ W_3^0 = 1$

$* \ W_3^1 = e^{-J\frac{2\pi}{3}} = \cos(120) - J\sin(120)$

$= -0.5 - J\frac{\sqrt{3}}{2}$

$* \ W_3^2 = e^{-J\frac{4\pi}{3}} = \cos(240) - J\sin(240)$

$= -0.5 + J\cdot\frac{\sqrt{3}}{2}$

$* \; W_3^4 = W_3^1 = -0.5 - J\frac{\sqrt{3}}{2}$;

$$[W_3] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5-J\frac{\sqrt{3}}{2} & -0.5+J\frac{\sqrt{3}}{2} \\ 1 & -0.5+J\frac{\sqrt{3}}{2} & -0.5-J\frac{\sqrt{3}}{2} \end{bmatrix}$$

$\boxed{Ex}$ for $x(n) = \{0.5, 1\}$ Determine 3-Point DFT.

For $N = 3$ $\quad X_3 = [W_3] \; x_3 \quad ; \; x_3 = \begin{bmatrix} 0.5 \\ 1 \\ 0 \end{bmatrix}$

$$X_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5-J\frac{\sqrt{3}}{2} & -0.5+J\frac{\sqrt{3}}{2} \\ 1 & -0.5+J\frac{\sqrt{3}}{2} & -0.5-J\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \\ 0 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 1.5 \\ -J\frac{\sqrt{3}}{2} \\ J\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$X(K) = \left\{ 1.5, -j\frac{\sqrt{3}}{2}, j\frac{\sqrt{3}}{2} \right\}$$

$$|X(K)| = \left\{ 1.5, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} \right\}$$

$$\angle X(K) = \left\{ 0, -90^\circ, 90^\circ \right\}$$